9/22/16

"Everyone has a gift, some people just never open theirs." -Mr. Callahan

HW: "Fractional Exponents" back #38, 43, 48, 54, 59, 61, 64, 67, 71, 75, 78

AIM: What are Complex Numbers?

Warm Up:

Definition: The imaginary unit, i, is defined as $\sqrt{-1}$. Therefore:

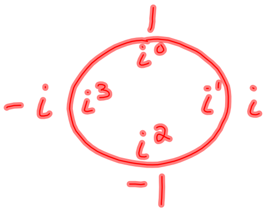
$i^0 = (\sqrt{-1})^0 = \boxed{1}$ $\qquad$ $i^4 = i^2 \cdot i^2 = (-1)(-1) = \boxed{1}$

$i^1 = \boxed{i}$ $\qquad$ $i^5 = i^2 \cdot i^3 = (-1)(-i) = \boxed{i}$

$i^2 = \sqrt{-1}\sqrt{-1} = \boxed{-1}$ $\qquad$ $i^6 = i^3 \cdot i^3 = (-i)(-i) = i^2 = \boxed{-1}$

$i^3 = i \cdot i^2 = i(-1) = \boxed{-i}$ $\qquad$ $i^7 = i^3 i^4 = (-i)(1) = \boxed{-i}$

We can easily simplify any power of i. We do this by:



(*) Divide the exponent by 4
whatever the remainder is
that is the new power

no decimal = 1
.25 = i
.5 = -1
.75 = $-i$

$\sqrt{x^2} = x$
$\sqrt{i^2} = i$
$\sqrt{-1} = \sqrt{i^2} = i$
$i^2 = -1$

Examples:

Simplify each.

1. $i^{20} = \boxed{1}$

$4\overline{)20}$ = 5 no decimal

3. i^{78}

$4\overline{)78}$ = 19.5 $= \boxed{-1}$

2. $i^{19} = \boxed{-i}$

$4\overline{)19}$ = 4.75

4. $3i^{11} \cdot 2i^5$

$4\overline{)11}$ = 2.75 $\qquad$ $i^{11} = -i$

$4\overline{)5}$ = 1.25 $\qquad$ $i^5 = i$

$3(-i)\ 2(i)$

$6(-i^2)$

$6(--1)$

$6(1)$

$\boxed{6}$

4) Alt

$3i^{11} \cdot 2i^5$ $\qquad$ $x^{11} \cdot x^5 = x^{16}$

$6i^{16}$

$4\overline{)16}$ = 4

no decimal

$6(1) = 6$

Property of negative square roots:

$$\sqrt{-c} = \sqrt{-1c} = \sqrt{-1}\sqrt{c} = i\sqrt{c}$$

⊛ Short cut take out negative and put "i"

Examples:

$\sqrt{-1} = \sqrt{i^2} = i$ skip to here

Simplify each.

5. $\sqrt{-25}$

$i\sqrt{25}$

$i\,5$

$\boxed{5i}$

6. $\sqrt{-32}$

$i\sqrt{32}$

$\sqrt{16}\,\sqrt{2}$

$i\,4\sqrt{2}$

$\boxed{4i\sqrt{2}}$

7. $-\sqrt{25} - \sqrt{-147}$

$-5 - i\sqrt{147}$

$\sqrt{49}\,\sqrt{3}$

$-i \cdot 7\sqrt{3}$

$\boxed{-5 - 7i\sqrt{3}}$

8. $\sqrt{-128}$

$i\sqrt{128}$

$\sqrt{64}\,\sqrt{2}$

$i\,8\sqrt{2}$

$\boxed{8i\sqrt{2}}$

9. $\sqrt{-9} + \sqrt{-16}$

$3i + 4i = \boxed{7i}$

Definition:

A number of the form $a+bi$, where a and b are real numbers and $i=\sqrt{-1}$ is called a **complex number.** a is called the **real part** and bi is called the **imaginary part**. A complex number written with the real part first and the imaginary part last is in **standard form.**

Examples:

Perform the operations and put your answers in standard form.

10. $(-1+2i)+(5-3i)$

combine like terms

$-1+5=4$ (Real)

$2i+(-3i)=-1i$ (Imaginary)

$\boxed{4-1i}$ or $\boxed{4-i}$ Complex number

11. $(-11-40i)-(2+10i)$

$-11-2=-13$

$-40i-10i=-50i$

$\boxed{-13-50i}$

12. $10i(6-8i)$

$i^2=-1$

$60i-80i^2$

$60i-80(-1)$

$60i+80$

$\boxed{80+60i}$

13. $(2+5i)(3-15i)$

$6\underbrace{-30i+15i}-75i^2$

$6-15i-75(-1)$

$6-15i+75$

$\boxed{81-15i}$

14. $\sqrt{-4}\cdot\sqrt{-10}\cdot\sqrt{36}$

15. $(5-\sqrt{-27})-(9+\sqrt{-108})$

16. $(-2+6i)(3-2i)$

17. $(4+i)(-5-3i)$

18. Simplify: $5i^{18}+7i^{25}+2i^{28}+6i^{43}$

19. Determine the result in simplest $a+bi$ form:

$$(5+2i)(-3+i)+4i(2+3i)$$

Definition:

$a+bi$ and $a-bi$ are called **complex conjugates.** So, $(a+bi)(a-bi)=$.